The Effect of Small-Scale Vertical Mixing of Horizontal Momentum in a General Circulation Model

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(Manuscript received 3 June 1974; in revised form 3 September 1974)

ABSTRACT

Several experiments are described in which the sub-grid-scale vertical eddy viscosity in the GISS global general circulation model was varied. The results show that large viscosities suppress large-scale eddies in middle and high latitudes, but enhance the circulation in the tropical Hadley cell and increase the extent of the tropical easterlies. Comparison with observations shows that the GISS model requires eddy viscosities $\sim$1 m^2/s or less to give realistic results for middle and high latitudes, and eddy viscosities $\sim$100 m^2/s to give realistic results for low latitudes. A plausible mechanism for the implied increase in small-scale mixing in low latitudes is cumulus convection.

1. Introduction

The theory of large-scale atmospheric motions has progressed to the point where it is now possible to give a qualitative picture of their role in the general circulation (Lorenz, 1967). Unfortunately the theory of small-scale motions has not progressed to a similar point, and this defect acts as a powerful limitation on the validity of numerical general circulation models. Such models can calculate the large-scale motions explicitly, but need to parameterize the effect of the small-scale motions (e.g., Manabe *et al.*, 1965; Washington and Kasahara, 1970; Arakawa, 1970). In the absence of sound theories the parameterizations are highly empirical and *ad hoc.*

One small-scale transport for which we have particularly little information is the vertical eddy transport of horizontal momentum. Direct measurements which indicate the magnitude of this eddy transport outside the boundary layer are lacking. Indirect information can be obtained from balance requirements. For example, Palmén and Newton (1969, Section 1.3) deduced that the vertical eddy flux of angular momentum in the Northern Hemisphere is downward and equal to about 8×10^4 kg/s^2 at the 500-mb level. This result suggests that the transport could be modeled by a diffusion law with an eddy viscosity. If we divide Palmén and Newton's value for the flux by a typical density of 0.7 kg/m^3, a typical axial distance of 4.5×10^6 m, and a typical shear of 1.5 $\times10^{-3}$ s^{-1}, we find for a typical value of this eddy viscosity 20 m^2/s.

However, this value represents the transport by both large-scale and small-scale motions, and therefore gives only an upper bound on the eddy viscosity for the small-scale motions. This upper bound is not very useful since one would expect the transport by large-scale eddies to be comparable to the above value. Mixing length theory suggests that the eddy viscosity arising from the large-scale tropospheric eddies should be of the order of the vertical velocity times the depth of the troposphere, and quasi-geostrophic theory suggests that the vertical velocity should be of the order of the Rossby number times the aspect ratio times the horizontal velocity. If we choose as typical a depth of 10^4 m, a Rossby number of 10^{-1}, an aspect ratio of 2×10^{-3}, and a horizontal velocity of 10 m/s, this prescription implies a value for the large-scale eddy viscosity $\sim$20 m^2/s. The small-scale eddy viscosity might be much less than this value.

Faced with this uncertainty, different modeling groups have generally settled for representing the sub-grid scale vertical transport of momentum by simple diffusion laws, but with quite different values for the eddy viscosity. For example, the GFDL model uses an eddy viscosity equal to zero above 2.5×10^3 m (Holloway and Manabe, 1971). The UCLA model employs a dynamic viscosity outside the boundary layer equal to 0.44 mb sec (Gates *et al.*, 1971), corresponding to an eddy viscosity of 70 m^2/s, in mid-troposphere. The NCAR model uses an eddy viscosity which is a function of Richardson number, but which is about 0.1 m^2/s in typically stable conditions outside the boundary layer (Washington and Kasahara, 1970).

Recently a general circulation model has been developed at the Goddard Institute for Space Studies (GISS), mainly for use in numerical experiments in

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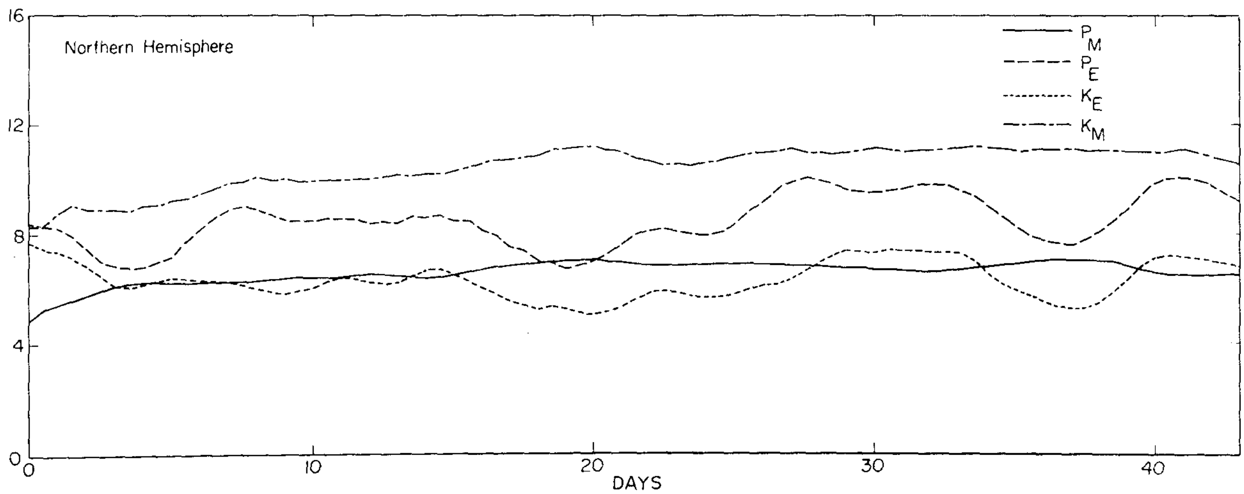
support of the Global Atmospheric Research Program. This model has been described by Somerville *et al.* (1974). Tests performed while the model was being developed showed that the calculated general circulation was sensitive to the strength of the sub-grid-scale vertical transport of momentum. Consequently, we undertook a series of numerical experiments in which we varied the strength of this transport in the GISS model. The resulting changes in the numerical results hopefully will yield insight into the role of this transport in the general circulation, and into how it might best be parameterized. This paper reports the results of these experiments.

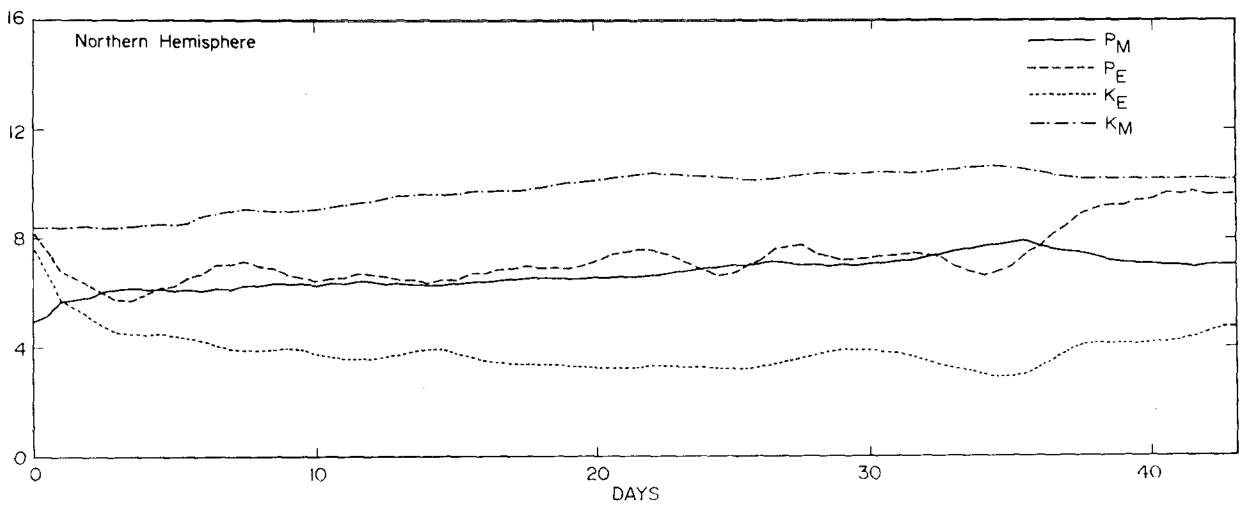
2. Experiments

The GISS model (Somerville *et al.*, 1974) is a global general circulation model based on the primitive equations of motion solved by finite-difference methods. The model includes a realistic distribution of continents (with topography), oceans and sea-surface temperature. The hydrologic cycle and major diabatic heat sources are parameterized. Precipitation and cloudiness occur in the model due to both grid-scale supersaturation and parameterized sub-grid-scale convection. The cloud and water vapor fields generated by the model are used in calculating solar and terrestrial radiative heating rates. Land temperature is determined from heat flux calculations, including a simplified ground hydrology. Surface fluxes and sub-grid-scale processes are treated parametrically using drag and eddy coefficients.

The model has 9 levels in the vertical equally spaced in sigma-coordinates, and a horizontal resolution of 4° in latitude and 5° in longitude. Therefore the eddy viscosities we will be discussing specifically represent transports in the free atmosphere by all motions with



(a)



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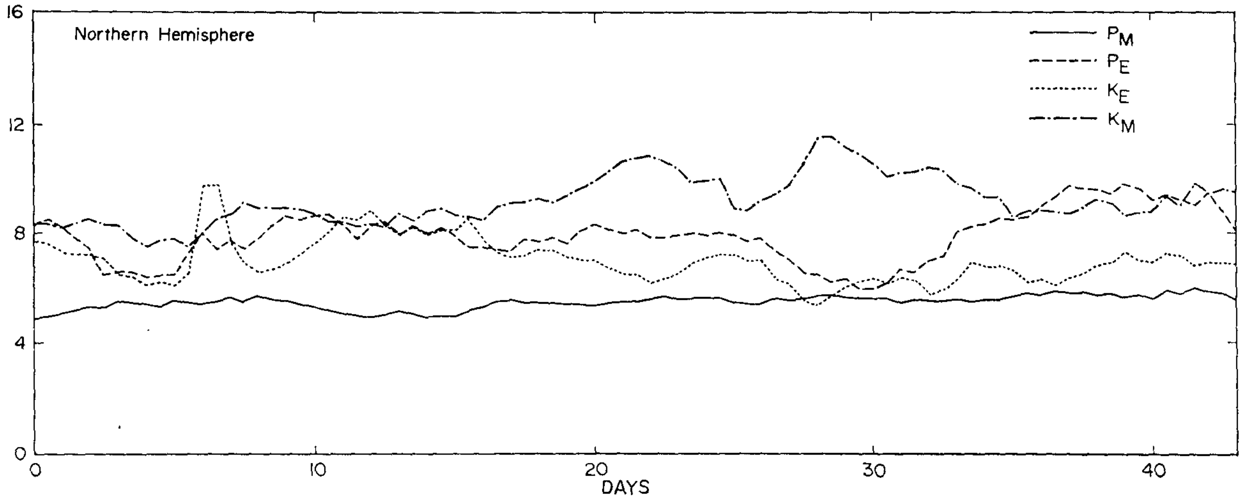

(c)

FIG. 1. Time evolution of tropospheric energy components. a) Experiment A (zero viscosity). b) Experiment B (large viscosity). c) Observations (calculated from NMC data). Units are 10^5 J/m², except for P_M which is 10^6 J/m².

scales too small to be resolved by these particular grid sizes. In the first two numerical experiments the sub-grid-scale vertical fluxes of zonal and meridional momentum, F_x and F_y, respectively, were given by

$$F_x = -\mu \frac{\partial u}{\partial z}, \quad F_y = -\mu \frac{\partial v}{\partial z}. \tag{2.1}$$

Here μ is the dynamic viscosity, u and v are the zonal and meridional wind components, respectively, and z is height. These two experiments differed only in the choice of μ. In experiment A, μ was chosen to be zero. In experiment B, μ was chosen to be 22 kg/m/s, corresponding to an eddy viscosity ranging from 20 m²/s near the ground to 70 m²/s near the tropopause. These values of μ are the most extreme values consistent with Palmén and Newton's (1969) value for the vertical flux of zonal momentum in mid-troposphere.

In addition to momentum diffusion between model layers, described by the above laws, the GISS model includes skin friction. This skin friction is parameterized by a simple drag law, and acts to decrease the velocities in the lowest model layer. This effect was not changed in any of our experiments. Thus in experiment A there is an exchange of momentum between the ground and the lowest model layer, but momentum can be exchanged with higher layers only by vertical advections explicitly calculated by the model.

The results of these two experiments, reported in the next section, led us to select a viscosity of 0.1 m²/s for the climatological experiment described by Somerville *et al.* (1974). We will designate that experiment as experiment C and also give some of its results for comparison with experiments A and B. In experiment C the kinematic viscosity rather than the dynamic viscosity was held constant, since it is difficult to justify an eddy stress per unit mass which is stronger near the tropopause than near the ground. Experiment C also differed from experiments A and B because of some slight changes in the parameterization of sub-grid-scale vertical diffusion of heat, but these changes had minor effects compared to the changes in eddy viscosity described above.

The experiments were all conducted in a manner identical to the climatological experiment described by Somerville *et al.* (1974). The initial condition was based on synoptic data for 20 December 1972, supplied by the National Meteorological Center (NMC). This realistic initial state allowed the integrations to approach a statistical equilibrium within 10 days. The integrations were carried out for 43 days and model-generated statistics were computed for days 13–43 inclusive, corresponding to January 1973. The eddy viscosity in experiment C, 0.1 m²/s, corresponds to a dissipative time scale of about 100 days for the troposphere, which is long compared to the 43-day integration period. Therefore the tropospheric statistics computed for experiment C should be essentially the same as those computed for the run with no viscosity, experiment A. In fact they are. Therefore experiment C serves primarily as a "predictibility" experiment. By comparing its results with those of experiment A, we get an idea of the GISS model's natural variability. The significant differences between experiments A and B reported in the next section are all many times larger than the differences between experiments A

TABLE 1. Average Northern Hemisphere tropospheric energetics for January 1973. Units are 10^5 J/m^2.

Source	K_M	P_M	K_E	P_E
Experiment A (zero viscosity)	10.8	68	6.3	8.6
Experiment B (high viscosity)	10.1	69	3.6	7.5
Experiment C (low viscosity)	10.7	62	6.2	8.5
Observations (NMC data)	9.5	55	6.9	8.0

and C. Statistics were also computed from observational data for comparison with the model-generated statistics.

3. The results

Figure 1 (a and b) shows the computed time evolution of different components of the atmosphere's energy integrated over the whole Northern Hemisphere troposphere (8 lowest model layers) in experiments A and B. K_M is the zonal kinetic energy, K_E the eddy kinetic energy, P_M the zonal available potential energy, and P_E the eddy available potential energy. The calculations of P_M and P_E allowed for variations in the static stability and therefore differ from the calculations for experiment C, given in Fig. 9 of Somerville *et al.* (1974).

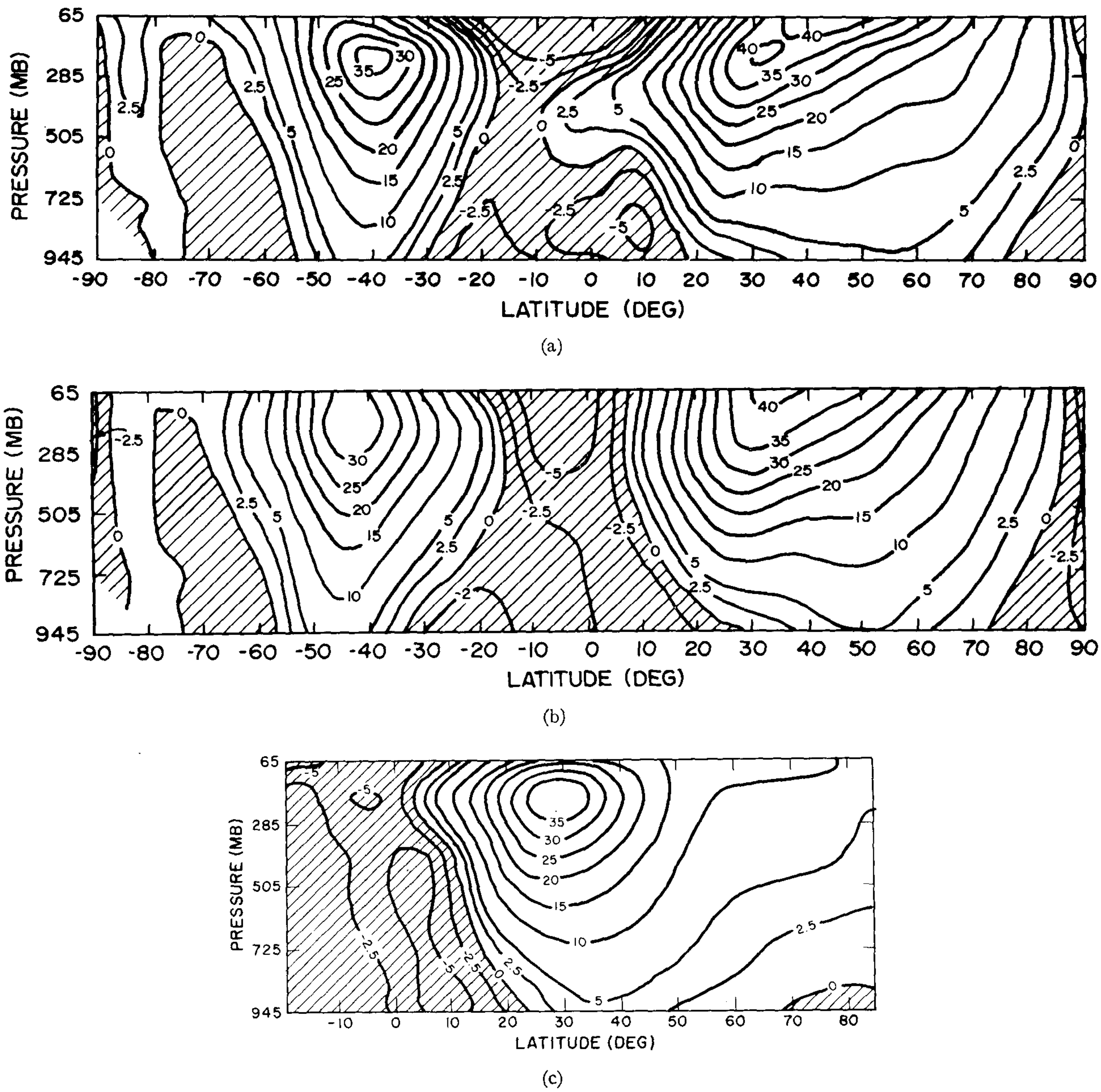

FIG. 2. January zonal mean field of zonal wind. Units are m/sec. a) Experiment A (zero viscosity). b) Experiment B (large viscosity) c) Observations (Oort and Rasmusson, 1971).

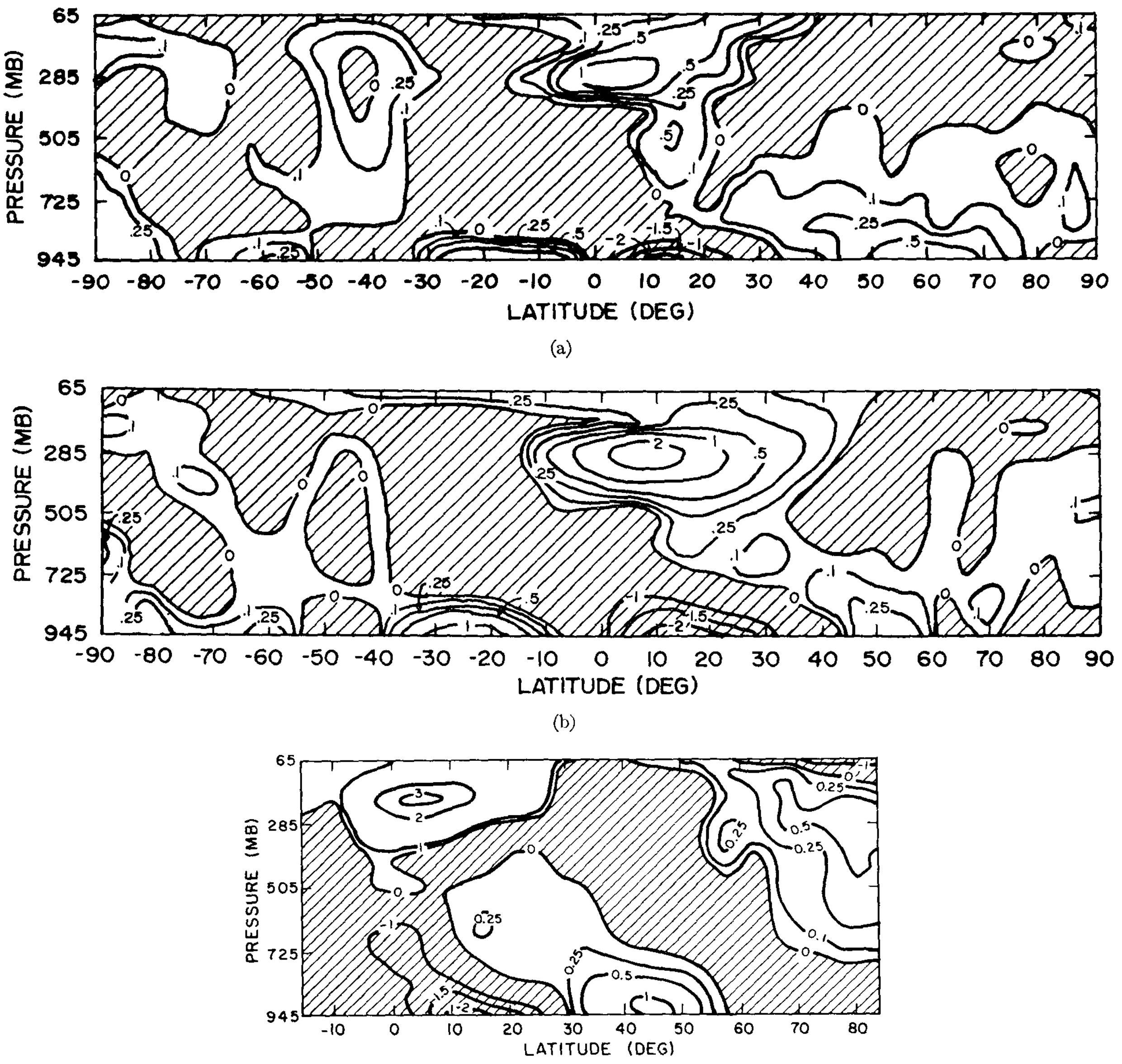

FIG. 3. January zonal mean field of meridional wind. Units are m/sec. a) Experiment A (zero viscosity). b) Experiment B (large viscosity). c) Observations (Oort and Rasmusson, 1971).

When P_M and P_E were re-calculated for experiment C in the same way as for experiments A and B, the results for experiment C were essentially the same as for experiment A. Figure 1 shows that the viscosity has only a small effect on the zonal energies. On the other hand it has a substantial effect on the eddy energies. The average level of K_E is greatly reduced by the large viscosity in experiment B, and the fluctuations in the eddy energies are almost eliminated. Figure 1c shows the evolution of the actual tropospheric energies for the same period, calculated from data supplied by NMC for December 1972 and January 1973. Comparing the figures we see that the zero- and low-viscosity experiments are much more realistic. Clearly, sub-grid-scale eddy viscosities of the order of 10 m²/s or larger lead to an unrealistic suppression of the index cycle and of the kinetic energy of large-scale eddies.

Table 1 shows the different energy components of the Northern Hemisphere troposphere averaged for all of January from the experiments and from the NMC data. The units are 10^5 J/m². Quantitatively, the viscosity in experiment B reduced the eddy kinetic energy by about 40% relative to experiments A and C. The reduction of K_E was caused directly by the sub-grid-scale viscosity: in experiment B the dissipation

TABLE 2. Maximum mean meridional velocity in the poleward branch of the tropical Hadley cell and the mass flux in the tropical cell for January.

Source	v (m/s)	ψ (10^{10} kg/s)
Experiment A (zero viscosity)	1.6	11
Experiment B (high viscosity)	2.8	19
Experiment C (low viscosity)	1.8	12
Observations (Oort & Rasmusson)	3.2	19

by internal friction was found to be comparable to that by surface friction, about 1.0 W/m². The reduction in K_E was also accompanied by a reduction in the eddy fluxes, and by a small reduction in the conversion of P_M to P_E, which led to a small reduction in the average level of P_E.

The above results indicate that only eddy viscosities on the order of 1 m²/s or smaller are consistent with the observed activity of atmospheric eddies. It was this result that led to the choice of a viscosity equal to 0.1 m²/s for the climatological experiment described by Somerville *et al.* (1974). In addition we found that the sub-grid-scale viscosity had a significant impact on the zonally averaged winds. Figure 2 shows the January mean zonal wind from experiments A and B. The high viscosity in experiment B substantially decreased the vertical shear of the zonal wind in tropical regions, thereby eliminating the tropical "bulge" of the Northern Hemisphere jet and broadening the region of easterlies in the tropics. Figure 2 also shows the January mean zonal wind from obsevations, taken from Oort and Rasmusson (1971). The zonal winds in experiment C were essentially identical to those in experiment A (see Somerville *et al.*, Fig. 14). However, the high viscosity experiment yielded more realistic zonal winds. In particular, the extent of the tropical easterlies is reproduced much more accurately in experiment B.

Figure 3 shows the January mean meridional wind from experiments A and B and from observations, the latter again taken from Oort and Rasmusson (1971). The results for experiment C are given in Fig. 15 of Somerville *et al.*, and are essentially the same as in experiment A. The high viscosity in experiment B substantially increased the strength of the tropical Hadley cell at the same time as it decreased the westerly zonal flow in the vicinity of the poleward branch of the Hadley cell. In fact the increase in the strength of the Hadley cell can be attributed to the decrease in the zonal flow. The conservation equation for momentum in the meridional direction is

$$\frac{dv}{dt} = -fu + \frac{\partial F_y}{\partial z} + G \qquad (3.1)$$

where d/dt is the total time derivative, f is the Coriolis parameter, and G is the pressure gradient term. Eq. (3.1) shows that a decrease in the zonal wind in the vicinity of the poleward branch of the Hadley cell leads to an acceleration of the poleward flow. This acceleration is the apparent cause of the increase in the Hadley circulation. Table 2 shows the maximum value of the mean meridional velocity in the poleward branch of the Hadley cell and the total mass flux in the tropical Hadley cell from the experiments and from observations. The latter are again taken from Oort and Rasmusson's results. Comparing the experiments with the observations we see that the high-viscosity experiment has the most realistic simulation of the meridional winds as well as of the zonal winds. The transports of zonal momentum, sensible heat, and specific humidity by the Hadley cell were also enhanced in the high viscosity experiment, and are in better agreement with the observations. Our results suggest that eddy viscosities of the order 100 m²/s are necessary near the tropopause to obtain realistic tropical mean wind fields.

Other model-generated statistics also show the effects of viscosity, although the effects are less pronounced than the ones cited above. In particular, the high-viscosity experiment had 20% more deep, penetrating cumulus convection and 20% more convective heating in the tropics than the zero-viscosity experiment. Associated with this increased convective activity was an increase in the static stability in tropical regions of 0.3 K/km, and an increase in the relative humidity near the tropical tropopause from 50% (in the zero-viscosity experiment) to 70% (in the high-viscosity experiment). The increased convective activity is apparently caused by increased convergence in the planetary boundary layer due to the large viscosity. The increased convective activity may contribute significantly to the enhancement of the Hadley circulation. Also the high-viscosity experiment had a weaker and less realistic Ferrel cell in mid-latitudes (cf. Figs. 3a and 3b).

4. Discussion

The above results appear at first sight to be paradoxical: large eddy viscosities are necessary for a realistic simulation of the zonal mean winds, and small eddy viscosities for a realistic simulation of the large-scale eddies. Closer examination removes the paradox: only the tropical zonal mean winds are appreciably improved by the large eddy viscosity, and the large-scale eddies are primarily confined to middle and high latitudes. The indicated conclusion is that the small-scale eddy viscosity is a strong function of latitude, with large values being appropriate in low latitudes and small values in middle and high latitudes. The specific values which give realistic results will depend on a particular model's characteristics, such as resolution. For the GISS model, values ~1 m²/s or smaller are appropriate in middle and high latitudes and values ~100 m²/s in low latitudes.

A related calculation has been reported by Holton and Colton (1972). They performed a diagnostic calculation for the 200-mb level in tropical regions and concluded that the mean vorticity field could be simulated correctly only by invoking an unusually large rate coefficient for viscous drag, D. This coefficient is defined in terms of the vertical fluxes of momentum by the relations

$$\frac{\partial F_x}{\partial z} = -Du$$
$$\frac{\partial F_y}{\partial z} = -Dv. \tag{4.1}$$

Holton and Colton deduced that $D \sim 1.5 \times 10^{-5}\ \mathrm{s}^{-1}$. They speculated that this apparent drag might be caused by deep cumulus towers mixing momentum. Colton (1973) extended this work by calculating the contribution to the apparent drag by the interaction of synoptic scale transient eddies, and found that this interaction could account for most of the apparent drag. Nevertheless, Colton had to retain a drag of strength $D = 0.25 \times 10^{-5}\ \mathrm{s}^{-1}$ to simulate the effect of smaller-scale motions.

Since the GISS model explicitly calculates the synoptic-scale interactions, the high viscosity $\sim 100\ \mathrm{m}^2/\mathrm{s}$ we found necessary to simulate the tropical circulations must be identified with the sub-synoptic scale motions. The corresponding rate coefficient for viscous drag at 200 mb may be estimated by dividing the eddy viscosity by the square of the height of the 200-mb level, $\sim 10^4$ m (cf. Eqs. 2.1 and 4.1). The result is $\sim 0.1 \times 10^{-5}\ \mathrm{s}^{-1}$, a value consistent with Colton's results. Our experiments indicate that even this small residual amount is important in simulating tropical circulations.

Holton and Colton's suggested mechanism, i.e., cumulus mixing, is a plausible source for this residual part of their drag coefficient. Such mixing will act primarily in low latitudes and will not produce as large viscosities in middle and high latitudes, consistent with our result that small viscosities are realistic in these latitudes. Independent evidence that vertical mixing of momentum by cumulus convection may play an important role in the general circulation has been presented by Houze (1973). This mechanism has not yet been included in general circulation models.

Acknowledgments. We are indebted to the National Meteorological Center for supplying us with data for December 1972 and January 1973.

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